HIGH SPEED AERODYNAMICS (R15A2108)

COURSE FILE

III B. Tech I Semester

(2018-2019)

Prepared By

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Department of Aeronautical Engineering

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

Affiliated to JNTU, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)

Maisammaguda, Dhulapally (Post Via. Kompally), Secunderabad – 500100, Telangana State, India.

MRCET VISION

- To become a model institution in the fields of Engineering, Technology and Management.
- To have a perfect synchronization of the ideologies of MRCET with challenging demands of International Pioneering Organizations.

MRCET MISSION

To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become pioneers of Indian vision of modern society.

MRCET QUALITY POLICY.

- To pursue continual improvement of teaching learning process of Undergraduate and Post Graduate programs in Engineering & Management vigorously.
- To provide state of art infrastructure and expertise to impart the quality education.

PROGRAM OUTCOMES
(PO's)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design / development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multi disciplinary environments.
12. **Life- long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

DEPARTMENT OF AERONAUTICAL ENGINEERING

VISION

Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation.

MISSION

The primary mission of the department is to promote engineering education and research. To strive consistently to provide quality education, keeping in pace with time and technology. Department passions to integrate the intellectual, spiritual, ethical and social development of the students for shaping them into dynamic engineers.

QUALITY POLICY STATEMENT

Impart up-to-date knowledge to the students in Aeronautical area to make them quality engineers. Make the students experience the applications on quality equipment and tools. Provide systems, resources and training opportunities to achieve continuous improvement. Maintain global standards in education, training and services.

PROGRAM EDUCATIONAL OBJECTIVES – Aeronautical Engineering

1. **PEO1 (PROFESSIONALISM & CITIZENSHIP):** To create and sustain a community of learning in which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.
2. **PEO2 (TECHNICAL ACCOMPLISHMENTS):** To provide knowledge based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.
3. **PEO3 (INVENTION, INNOVATION AND CREATIVITY):** To make the students to design, experiment, analyze, and interpret in the core field with the help of other multi disciplinary concepts wherever applicable.
4. **PEO4 (PROFESSIONAL DEVELOPMENT):** To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.
5. **PEO5 (HUMAN RESOURCE DEVELOPMENT):** To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

1. To mould students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.
2. To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.
3. Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments in the field of Aeronautical Engineering.
4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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III Year B. Tech, ANE-I Sem	**5**	**1/-/-**	**4**

(R15A2108) HIGH SPEED AERODYNAMICS

Objectives:

- Study the basic governing equations of compressible flows and its parameters.
- Study the effects of Shock and Expansion waves on aerodynamic characteristics.
- Learn about the experimental methods to study about compressible flows.

Tables: Isentropic, 1D Flow With Heat Addition and Friction, Normal Shock, Oblique Shock.

UNIT-I ONE DIMENSIONAL COMPRESSIBLE FLOWS

Review of Thermodynamics. Definition of Compressibility, Review of Governing equations. Stagnation conditions, Speed of sound, Mach number, flow regimes, shock waves. Alternative forms of Energy equations, Normal shock relations, Hugonoit equation, One dimensional flow with heat addition and one dimensional flow with friction.

UNIT-II OBLIQUE SHOCK AND EXPANSION WAVES

Oblique shock relations. Super sonic flow over a wedge Θ - β - M relations strong and weak shock solutions , Shock polar. Regular reflection from a solid boundary. Pressure deflection diagrams, Intersections of shock wave. Expansion waves. Prandtl – Meyer Expansion. Shock Expansion theory. Detached shock in front of blunt body.

UNIT-III

SUBSONIC COMPRESSIBLE AND SUPERSONIC LINEARISED FLOW OVER AIRFOIL

Introduction - Velocity potential equation –small perturbation equation - Prandtl-Glauert compressibility corrections - Critical Mach number - Drag divergence Mach number - Area rule - Supercritical airfoil. Linearized supersonic pressure coefficient- Improved compressibility correction factors, Application to airfoil. conical flows-physical aspects, Delta Wing Aerodynamics.

UNIT- IV

FLOW THROUGH NOZZLES AND VARIABLE AREA DUCTS

Area-velocity relation, Isentropic flow through Convergent – Divergent nozzles. Choked flow conditions. Normal shock. Under and Over expansion conditions. Flow through diffusers – wave reflections from a free boundary. Method of Characteristics Application to supersonic wind tunnels and rocket engine.

UNIT-V EXPERIMENTAL AERODYNAMICS

Model testing in wind tunnels and types of wind tunnels. Pressure, Temperature, Velocity measurements – Hotwire and Laser – Doppler anemometer. Force measurements – Wind tunnel balances. Scale effects and corrections, wall interferences, Flow visualization techniques-schileren and shadowgraph methods.

Text Books:

1. Anderson, J .D., Fundamental of Aerodynamics, Mc Graw-Hill International third edition Singapore-2001.
2. Anderson, J .D., Modern Compressible Flow with Historical Perspective, Mc Graw-Hill International third edition Singapore-2004.
3. W.E. Rae & Allen Pope, Low speed wind tunnel testing, John Willey &sons

Reference Books:

1. Radhakrishnan, E, E., Gas Dynamics, Prentice Hall of India, 1995.
2. Hodge B.K & Koenig K Compressible Fluid Dynamics with Computer Application, Prentice Hall, 1995
3. Clancy, L.J., Aerodynamics, Pitman, 1986, Macmillan, 1985

Outcomes:

- Understand the compressible flow parameters effecting flow behavior.
- Able to design nozzle, diffuser and variable area ducts to obtain required aerodynamic outputs.
- Able to understand experimental techniques to find aerodynamic characteristics of different geometries.

UNIT-I
ONE DIMENSIONAL COMPRESSIBLE FLOWS

Review of Thermodynamics

Perfect gas:

A perfect gas is one whose individual molecules interact only via direct collisions, with no other intermolecular forces present. For such a perfect gas, the properties p, ρ, and the temperature T are related by the following equation of state

$p = \rho RT$

where R is the specific gas constant. For air, R = 287J/kg-K◦. It is convenient at this point to define the specific volume as the limiting volume per unit mass,

$$v \equiv \lim_{\Delta\mathcal{V}\to 0} \frac{\Delta\mathcal{V}}{\Delta m} = \frac{1}{\rho}$$

which is merely the reciprocal of the density. In general, the nomenclature "specific X" is synonymous with "X per unit mass". The equation of state can now be written as

$p\upsilon = RT$

which is the more familiar thermodynamic form.

The appearance of the temperature T in the equation of state means that it must vary within the flowfield. Therefore, T(x, y, z) must be treated as a new field variable in addition to ρ(x, y, z). In the moving CV scenario above, the change in the CV's volume is not only accompanied by a change in density, but by a change in temperature as well.

The appearance of the temperature also means that thermodynamics will need to be addressed. So in addition to the conservation of mass and momentum which were employed in low speed flows, we will now also need to consider the conservation of energy. The following table compares the variables and equations which come into play in the two cases.

	Incompressible flow	Compressible flow
Variables:	V, p	V, p, ρ, T
Equations:	mass, momentum	mass, momentum, energy, state

Thermodynamics Concepts: System

A thermodynamic system is defined as a definite quantity of matter or a region in space upon which attention is focussed in the analysis of a problem. We may want to study a quantity of matter contained with in a closed rigid walled chambers, or we may want to consider something such as gas pipeline through which the matter flows. The composition of the matter inside the system may be fixed or may change through chemical and nuclear reactions. A system may be arbitrarily defined. It becomes important when exchange of energy between the system and the everything else outside the system is considered. The judgement on the energetics of this exchange is very important.

Surroundings

Everything external to the system is surroundings. The system is distinguished from its surroundings by a specified boundary which may be at rest or in motion. The interactions between a system and its surroundings, which take place across the boundary, play an important role in thermodynamics. A system and its surroundings together comprise a universe.

Types of systems

Two types of systems can be distinguished. These are referred to, respectively, as closed systems and open systems or control volumes. A closed system or a control mass refers to a fixed quantity of matter, whereas a control volume is a region in space through which mass may flow. A special type of closed system that does not interact with its surroundings is called an **Isolated system**.

Two types of exchange can occur between the system and its surroundings:

1. energy exchange (heat or work) and
2. Exchange of matter (movement of molecules across the boundary of the system and surroundings).

Based on the types of exchange, one can define

- **isolated systems:** no exchange of matter and energy
- **closed systems:** no exchange of matter but some exchange of energy
- **open systems:** exchange of both matter and energy

If the boundary does not allow heat (energy) exchange to take place it is called adiabatic boundary otherwise it is diathermal boundary.

<u>Laws of thermodynamics</u>

<u>1.2.1</u> **<u>Zeroth law of thermodynamics:</u>** This law states that 'when system A is in thermal equilibrium with system B and system B is separately in thermal equilibrium with system C then system A and C are also in thermal equilibrium'.

This law portrays temperature as a property of the system and gives basis of temperature measurement.

<u>1.2.2</u> **<u>First law of thermodynamics:</u>** It states the energy conservation principle, 'energy can neither be created nor be destroyed but one form of the energy can be converted to other'.

Implimentation of first law for a thermodynamic process defines 'internal energy' as a property of the system. According to this law for a closed system, when some amount of heat is supplied to the system, part of it is used to convert into work and rest is stored in the system in the form of internal energy. For the open system, heat supplied splits into enthalpy change and work done by the system.

First law of thermodynamics is thus associated with a corollary which states that there can be no machine (Perpetual Motion Machine of first kind) which can produce continuous work output without having any heat interaction with the surrounding.

Internal Energy and Enthalpy

The law of conservation of energy involves the concept of internal energy, which is the sum of the energies of all the molecules of a system. In fluid mechanics we employ the specific internal energy, denoted by e, which is defined for each point in the flowfield. A related quantity is the specific enthalpy, denoted by h, and related to the other variables by

$h = e + p\upsilon$

The units of e and h are $(\text{velocity})^2$, or m^2/s^2 in SI units.

For a calorically perfect gas, which is an excellent model for air at moderate temperatures, both e and h are directly proportional to the temperature. Therefore we have
where c_v and c_p are specific heats at constant volume and constant pressure, respectively. h

$- e = p\upsilon = (c_p - c_v)T$

and comparing to the equation of state, we see

that $c_p - c_v = R$

Defining the ratio of specific heats, $\gamma \equiv c_p/c_v$, we can with a bit of algebra write

$$c_v = \frac{1}{\gamma - 1} R$$
$$c_p = \frac{\gamma}{\gamma - 1} R$$

so that c_v and c_p can be replaced with the equivalent variables γ and R. For air, it is handy to remember that

$$\gamma = 1.4 \qquad \frac{1}{\gamma - 1} = 2.5 \qquad \frac{\gamma}{\gamma - 1} = 3.5 \qquad \text{(air)}$$

Isentropic relations:

Isentropic relations are the relations between thermodynamic properties if the system undergoes isentropic process.

Consider a closed system interacting dQ amount of energy with the surrounding. If dU is the change in internal energy of the system and pdV is work done by the system against pressure p due to volume change dV. According to First Law of Thermodynamics we know that,

$dQ = dU + pdV$

From Second law of Thermodynamics,

$$\frac{dQ}{T} = dS$$

$dQ = TdS$

Here dS is the entropy change due to reversible heat interaction dQ.
Therefore, combining First and Second Laws of Thermodynamics,

$$TdS = dU + pdV \qquad (1.1)$$

However, we know that, if H is enthalpy of the system then, H = U + pV

dH = dU + pdV + VdP

dH - Vdp = dU + pdV (1.2)

Combining equations 1.1 and 1.2 we get,

TdS = dH - Vdp

For system with unit mass of matter, above equation can be written as,

Tds = dh – vdp (1.3)

Here s, h and *v* are specific entropy, enthalpy and volume respectively.

Speciality of equation 1.3 is, its usefulness to calculate entropy change of any reversible process as below,

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{v}{T}dp$$

$$ds = C_p\frac{dT}{T} - R\frac{dp}{P}$$

where, dh = Cp dT and for calorifically perfect gas Cp (Specific heat at constant pressure) is assumed to be constant,

Integrating above equation from starting state 1 to end state 2 of the process which system has undergone

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

For the special case, if system undergoes reversible adiabatic or isentropic process then, entropy change of the system (ds) is zero. Therefore equation 1.3 can be written as,

$C_pdT = vdp$

From ideal gas realation, *v* = RT/P, above equations becomes

$$C_pdT = \frac{RT}{P}dp$$

$$\frac{C_pdT}{RT} = \frac{dp}{P} \qquad (1.4)$$

We know the relations between specific heats as

$$\therefore\ C_p - C_v = R$$

$$\therefore\ C_p - \frac{C_p}{\gamma} = R$$

$$C_p = \frac{\gamma R}{\gamma - 1} \qquad C_v = \frac{R}{\gamma - 1}$$

Substituing above expression for Cp in equation (1.4), we get

$$\frac{\gamma}{\gamma-1}\frac{dT}{T}=\frac{dp}{P}$$

Integrating above equation from state 1 to state 2 for the isentropic process of the system, we get,

$$\frac{\gamma}{\gamma-1}\ln\left(\frac{T_2}{T_1}\right)=\ln\left(\frac{P_2}{P_1}\right)$$

$$\left(\frac{P_2}{P_1}\right)=\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (1.5)$$

Since the system is undergoing adiabatic process from state 1 to state 2,

$$\frac{P}{\rho^{\gamma}}=Const.$$

$$\left(\frac{\rho_1}{\rho_2}\right)=\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} \qquad (1.6)$$

Here relations 1.5 and 1.6 are called as Isentropic equations.

Compressibility of fluid and flow

A compressible flow is a flow in which the fluid density ρ varies significantly within the flowfield. Therefore, ρ(x, y, z) must now be treated as a field variable rather than simply a constant. Typically, significant density variations start to appear when the flow Mach number exceeds 0.3 or so. The effects become especially large when the Mach number approaches and exceeds unity.

$$\text{Compressibility } (\tau)=-\frac{1}{\vartheta}\frac{\partial\vartheta}{\partial P}=\frac{1}{\rho}\frac{\partial\rho}{\partial P}$$

Compressibility is thus inverse of bulk modulus. Hence compressibility can be defined as the incurred volummetric strain for unit change in pressure. Negative sign in the above expression is the fact that volume decreases with increase in applied pressure. For example, air is more compressible than water. Since definition of compressibility involves change in volume due to change in pressure, hence compressibility can be isothermal, where volume change takes place at constant temperature or isentropic where volume change takes place at constant entropy.

$$\text{Isothermal compressibility } (\tau)=-\frac{1}{\vartheta}\left(\frac{\partial\vartheta}{\partial P}\right)_{T=\text{constant}}=\frac{1}{\rho}\left(\frac{\partial\rho}{\partial P}\right)_{T=\text{constant}}$$

$$\text{Isentropic compressibility } (\tau)=-\frac{1}{\vartheta}\left(\frac{\partial\vartheta}{\partial P}\right)_{s=\text{constant}}=\frac{1}{\rho}\left(\frac{\partial\rho}{\partial P}\right)_{s=\text{constant}}$$

Also, for isothermal compressibility we know,

$$\tau = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{T=\text{constant}}$$

Since $P\vartheta = RT$ for ideal gas, we have,

$$\left(\frac{\partial \vartheta}{\partial P}\right)_{T=\text{constant}} = -\frac{\vartheta}{P}$$

Hence, isothermal compressibility is

$$\tau = \frac{1}{P} \qquad (1.7)$$

For isentropic compressibility we know,

$$\tau = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{s=\text{constant}}$$

Since, PV^{γ} = constant for isentropic process for an ideal gas, we have,

$$\left(\frac{\partial \vartheta}{\partial P}\right)_{s=\text{constant}} = -\frac{\vartheta}{\gamma P}$$

Hence, isentropic compressibility is

$$\tau = \frac{1}{\gamma P}$$

Comparing equations (1.7) and (1.8) we can see that, isothermal compressibility is always higher than isentropic compressibility of gas since specific heat ratio is always greater than one. This inturn means that it is simpler to change the volume of a gas isothermally than isentropically. In otherwards, it means that we need lesser amount of pressure to bring a particular amount of change in volume during isothermal process than during isentropic process.

Fluid flow is said to be compressible if density of the fluid changes roughly 5% of its original density during its flow.

$$d\rho = \rho \times \tau \times dP$$

From this relation it is very clear that, percenetage change in density of fluid flow will be higher if either compressibility of the fluid is higher or pressure difference is high. Hence, compressible fluids exposed to smaller pressure difference situations can exhibit incompressible flow and at the same time incompressible fluids exposed to high pressure difference situations can exhibit compressible flow.

Review of Governing equations

Continuity equation

Physical principle – Mass can be neither created nor destroyed.

$$-\oiint_S \rho \mathbf{V} \cdot d\mathbf{S} = \frac{\partial}{\partial t} \oiiint_{\mathscr{V}} \rho \, d\mathscr{V}$$

Momentum equation

$$\oiint_S (\rho \mathbf{V} \cdot d\mathbf{S})\mathbf{V} + \oiiint_{\mathscr{V}} \frac{\partial(\rho \mathbf{V})}{\partial t} d\mathscr{V} = \oiiint_{\mathscr{V}} \rho \mathbf{f} \, d\mathscr{V} - \oiint_S p \, d\mathbf{S}$$

Energy equation

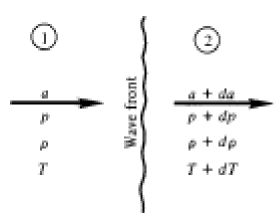
$$\oiiint_{\mathscr{V}} \dot{q}\rho \, d\mathscr{V} - \oiint_S p\mathbf{V} \cdot d\mathbf{S} + \oiiint_{\mathscr{V}} \rho(\mathbf{f} \cdot \mathbf{V}) \, d\mathscr{V} = \oiiint_{\mathscr{V}} \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] d\mathscr{V} + \oiint_S \rho\left(e + \frac{V^2}{2}\right)\mathbf{V} \cdot d\mathbf{S}$$

Stagnation conditions

In fluid dynamics, a **stagnation point** is a point in a flow field where the local velocity of the fluid is zero. If flow is brought to rest adiabatically then temperature at this point is known as total temperature (T_O) and enthalpy is known as total enthalpy (h_o) and if flow is brought to rest isentropically then pressure at this point is known as total pressure (P_O) and density is known as total density (ρ_o).

Speed of sound, Mach number,

In air molecules collide each other transfer energy as wave and also cause pressure to change slightly. Speed of such wave is calculated by considering wave is moving with velocity *a* in air.



Applying continuity equation to above flow model gives

$$\rho a = (\rho + d\rho)(a + da)$$

$$\rho a = \rho a + a \, d\rho + \rho \, da + d\rho \, da$$

Solving above equation gives

$$a = -\rho \frac{da}{d\rho}$$

Applying momentum equation to above flow model gives

$$p + \rho a^2 = (p + dp) + (\rho + d\rho)(a + da)^2$$

$$dp = -2a\rho \, da - a^2 \, d\rho$$

$$da = \frac{dp + a^2 \, d\rho}{-2a\rho}$$

$$a = -\rho \left[\frac{dp/d\rho + a^2}{-2a\rho} \right]$$

$$a^2 = \frac{dp}{d\rho}$$

Speed of sound is given by

$$\boxed{a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{\frac{v}{\tau_s}}}$$

For isothermal flow, Speed of sound is given by

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma p}{\rho}$$

$$\boxed{a = \sqrt{\frac{\gamma p}{\rho}}}$$

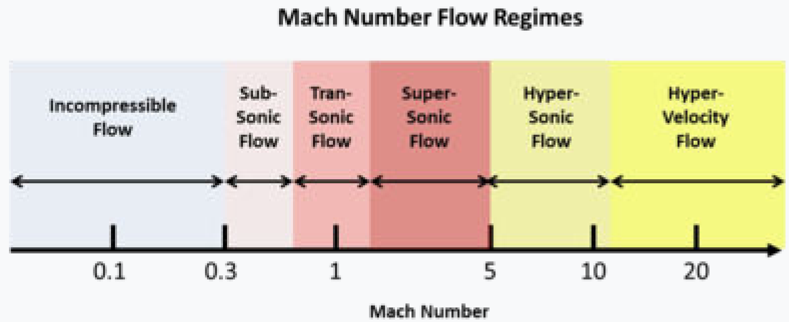
For a calorifically perfect gas Speed of sound is given by

$$\boxed{a = \sqrt{\gamma R T}}$$

$$\frac{V^2/2}{e} = \frac{V^2/2}{c_v T} = \frac{V^2/2}{RT/(\gamma - 1)} = \frac{(\gamma/2)V^2}{a^2/(\gamma - 1)} = \frac{\gamma(\gamma - 1)}{2} M^2$$

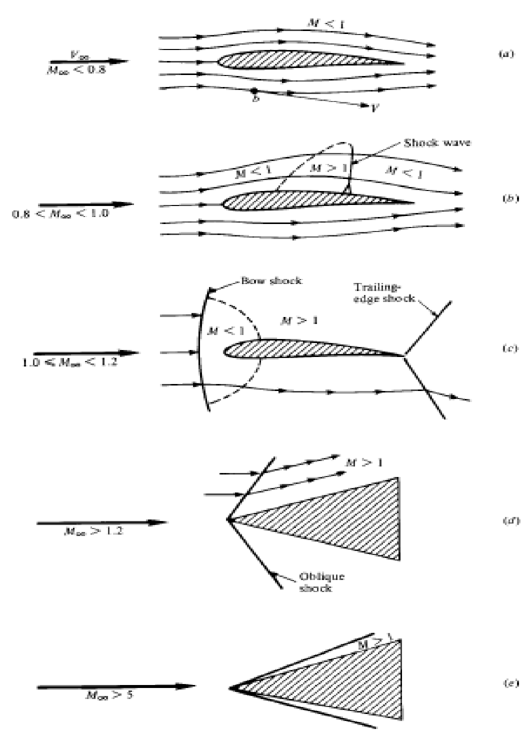
Flow regimes

The Mach number (M) is defined as the ratio of the speed of an object (or of a flow) to the speed of sound. For instance, in air at room temperature, the speed of sound is about 340 m/s (1,100 ft/s). M can range from 0 to ∞, but this broad range falls naturally into several flow regimes. These regimes are subsonic, transonic, supersonic, hypersonic, and hypervelocity flow. The figure below illustrates the Mach number "spectrum" of these flow regimes.



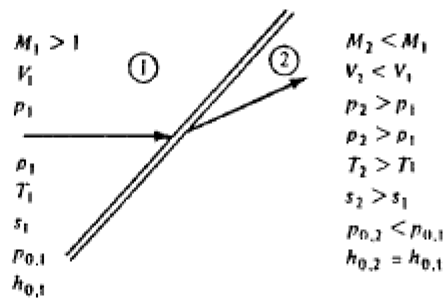
Mach number flow regimes spectrum

These flow regimes are not chosen arbitrarily, but rather arise naturally from the strong mathematical background that underlies compressible flow (see the cited reference textbooks). At very slow flow speeds the speed of sound is so much faster that it is mathematically ignored, and the Mach number is irrelevant. Once the speed of the flow approaches the speed of sound, however, the Mach number becomes all-important, and shock waves begin to appear. Thus the transonic regime is described by a different (and much more difficult) mathematical treatment. In the supersonic regime the flow is dominated by wave motion at oblique angles similar to the Mach angle. Above about Mach 5, these wave angles grow so small that a different mathematical approach is required, defining the Hypersonic speed regime. Finally, at speeds comparable to that of planetary atmospheric entry from orbit, in the range of several km/s, the speed of sound is now comparatively so slow that it is once again mathematically ignored in the Hypervelocity regime.

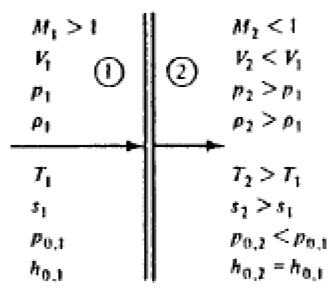


Shock waves

A **shock wave** (also spelled **shockwave**), or **shock**, is a type of propagating disturbance. It is a thin layer of 10^{-5} cm thickness. When a wave moves faster than the local speed of sound in a fluid, it is a shock wave. Like an ordinary wave, a shock wave carries energy and can propagate through a medium; however, it is characterized by an abrupt, nearly discontinuous change in pressure, temperature and density of the medium.

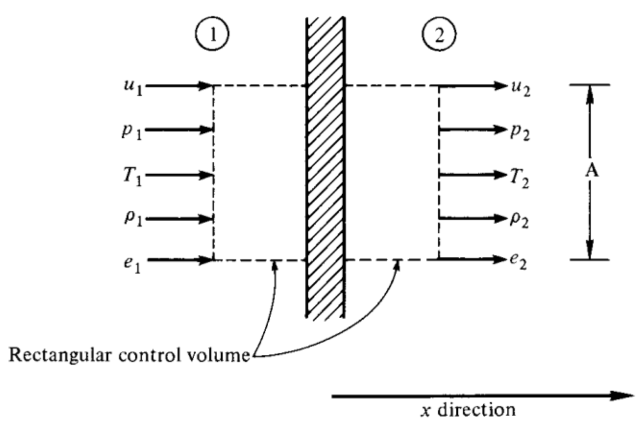


(a) Oblique shock wave



(b) Normal shock wave

One Dimensional Flow Equations



Consider control volume as shown in figure above and flow properties velocity u, pressure p, temperature T, density ρ and enthalpy h changes from suffix 1 to 2. Applying continuity, momentum and energy equations to above control volume yields the following one dimensional fluid flow governing equations:

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

Alternative forms of Energy equations

From the one dimensional energy equation for adiabatic flow no heat addition changes to

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

For a calorically perfect gas enthalpy h=C_pT then above equation changes to

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}$$

$$\frac{\gamma R T_1}{\gamma - 1} + \frac{u_1^2}{2} = \frac{\gamma R T_2}{\gamma - 1} + \frac{u_2^2}{2}$$

Substituting speed of sound equation then

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}$$

$$\frac{\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1}\right) + \frac{u_1^2}{2} = \frac{\gamma}{\gamma - 1}\left(\frac{p_2}{\rho_2}\right) + \frac{u_2^2}{2}$$

Consider the fluid is brought to Mach 1 at point 2 then flow is sonic in region 2 and suffix 2 is replaced with prefix * in equations representing sonic conditions.

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{a^{*2}}{2}$$

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2}$$

Now consider fluid is brought to rest isentropically i.e u_2=0, at point representing total conditions denoted by suffix o then equation changes to

$$c_p T + \frac{u^2}{2} = c_p T_o$$

$$\frac{T_o}{T} = 1 + \frac{u^2}{2c_p T} = 1 + \frac{u^2}{2\gamma RT/(\gamma - 1)} = 1 + \frac{u^2}{2a^2/(\gamma - 1)} = 1 + \frac{\gamma - 1}{2}\left(\frac{u}{a}\right)^2$$

$$\boxed{\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2}$$

$$\frac{p_o}{p} = \left(\frac{\rho_o}{\rho}\right)^{\gamma} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma - 1)}$$

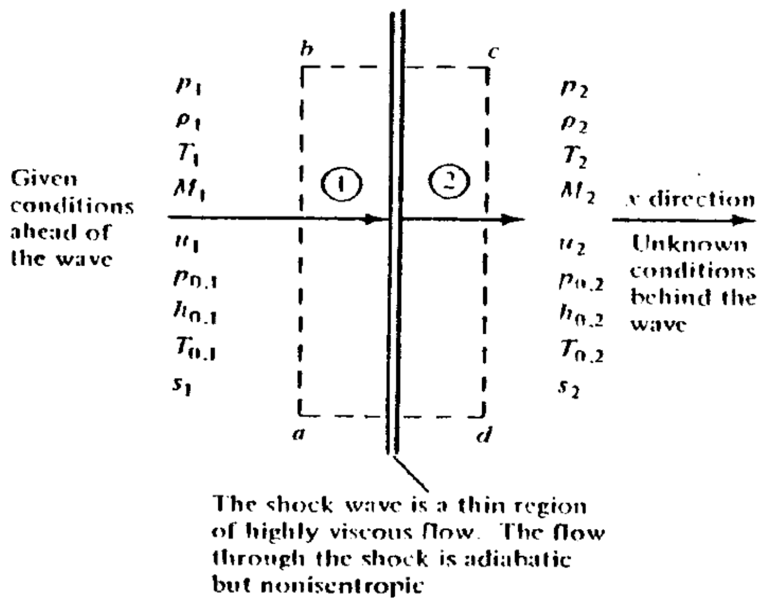
$$\boxed{\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)}}$$

$$\boxed{\frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma - 1)}}$$

The above equations give the relation between total to static conditions.

Normal shock relations

A shock which is perpendicular to flow direction is known as normal shock. It commonly occurs in supersonic flow changes the upstream supersonic flow to subsonic.



Applying one dimensional fluid flow governing equation to flow across normal shock by considering a control volume around it as shown in above fig.

$$\rho_1 u_1 = \rho_2 u_2 \qquad \text{(continuity)}$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad \text{(momentum)}$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad \text{(energy)}$$

$$p = \rho RT$$

$$h = c_p T$$

Using continuity equation in momentum equation gives

$$\frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = u_2 - u_1$$

Substituting speed of sound in above equation gives

$$\frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2} = u_2 - u_1$$

From alternative forms of energy equations

$$a_1^2 = \frac{\gamma+1}{2} a^{*^2} - \frac{\gamma-1}{2} u_1^2$$

$$a_2^2 = \frac{\gamma+1}{2} a^{*^2} - \frac{\gamma-1}{2} u_2^2$$

$$\frac{\gamma+1}{2}\frac{a^{*^2}}{\gamma u_1} - \frac{\gamma-1}{2\gamma} u_1 - \frac{\gamma+1}{2}\frac{a^{*^2}}{\gamma u_2} + \frac{\gamma-1}{2\gamma} u_2 = u_2 - u_1$$

Dividing by u_2-u_1 to above equation gives

$$\frac{\gamma+1}{2\gamma u_1 u_2} a^{*^2} + \frac{\gamma-1}{2\gamma} = 1$$

Solving for a* gives $\boxed{a^{*^2} = u_1 u_2}$

This equation is known as ***Prandtl relation*** which is very useful for normal shock relations.

$$1 = \frac{u_1}{a^*}\frac{u_2}{a^*} = M_1^* M_2^*$$

$$\boxed{M_2^* = \frac{1}{M_1^*}}$$

Thus above equation concludes the flow behind normal shock is always subsonic (M_2) for given supersonic inflow of M_1

$$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

$$\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2} = \left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]^{-1}$$

$$\boxed{M_2^2 = \frac{1+[(\gamma-1)/2]M_1^2}{\gamma M_1^2 - (\gamma-1)/2}}$$

The above relation is used to find the subsonic flow Mach number downstream normal shock. Similarly other flow properties are calculated using following relations. Density relation is obtained by using continuity equation as follows:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_2 u_1} = \frac{u_1^2}{a^{*^2}} = M_1^{*^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

From momentum equation, we have

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2$$

$$p_2 - p_1 = \rho_1 u_1(u_1 - u_2) = \rho_1 u_1^2\left(1 - \frac{u_2}{u_1}\right)$$

$$\frac{p_2 - p_1}{p_1} = \gamma M_1^2\left(1 - \frac{u_2}{u_1}\right)$$

Substituting above density relation in above equation gives pressure relation across normal shock

$$\frac{p_2 - p_1}{p_1} = \gamma M_1^2\left[1 - \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

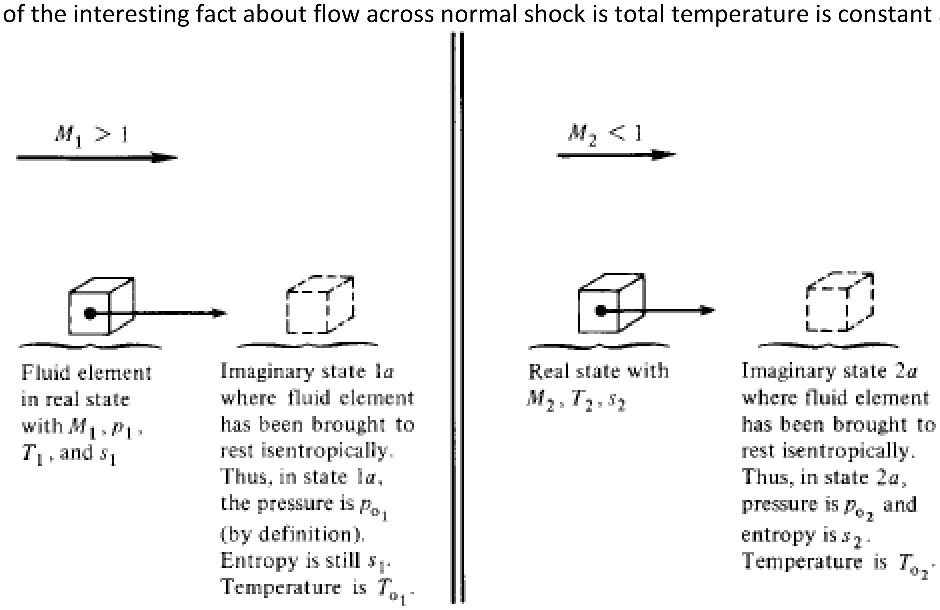
$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

From perfect gas equation the change temperature across shock is obtained as follows:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right)$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right]\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

One of the interesting fact about flow across normal shock is total temperature is constant across normal shock



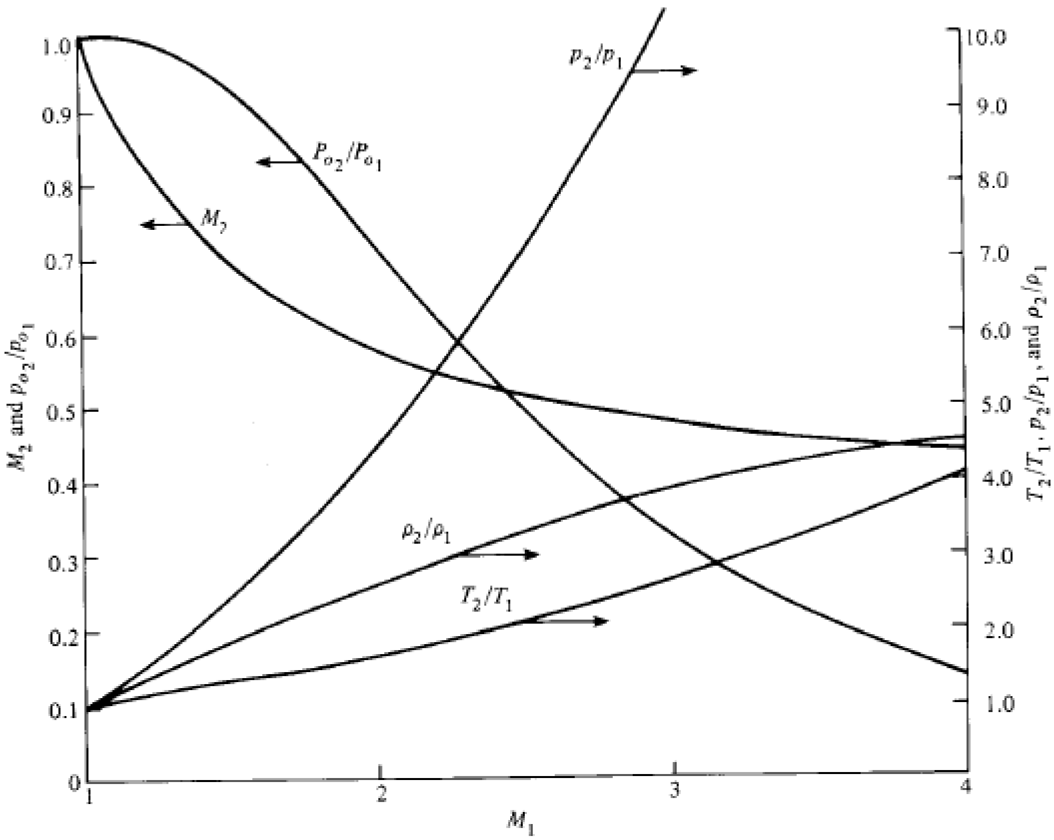
$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2}$$

$$c_p T_o = c_p T + \frac{u^2}{2}$$

$$c_p T_{o_1} = c_p T_{o_2}$$

$$\boxed{T_{o_1} = T_{o_2}}$$

By using above relations the change in flow properties across normal shock is shown graphical in the below figure.



Hugonoit equation

The normal shock relations gives the changes in terms of velocity, pressure etc. However the static pressure always increases across shock which acts itself as a thermodynamic device like compressing gas. Huganiot equation is derives to express the changes across normal shock in terms of pure thermodynamic variables as follows:

From continuity equation

$$u_2 = u_1\left(\frac{\rho_1}{\rho_2}\right)$$

Substituting in momentum equation

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2\left(\frac{\rho_1}{\rho_2}u_1\right)^2$$

Solving for velocity u_1,

$$u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right)$$

$$u_1 = u_2\left(\frac{\rho_2}{\rho_1}\right)$$

Similarly to get velocity u_2,

$$u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)$$

Considering one dimensional energy equation

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

and recalling that by definition $h = e + p/\rho$, we have

$$e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2}$$

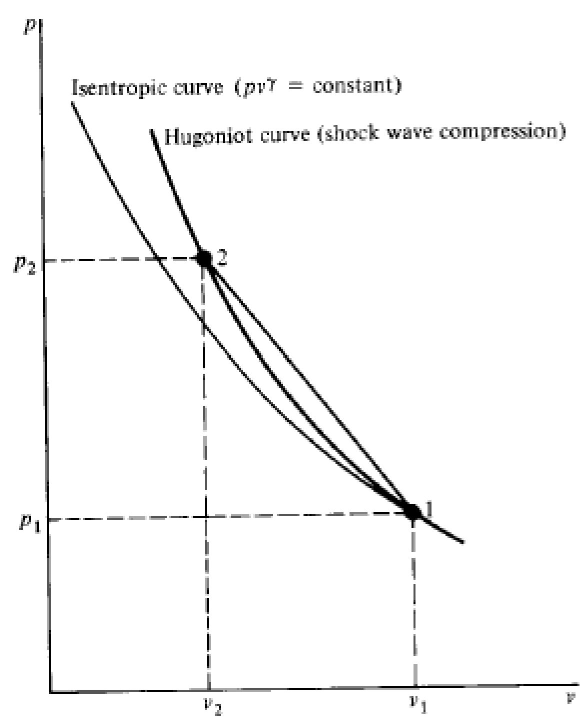
Substituting velocities in above equation

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right)\right] = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)\right]$$

$$e_2 - e_1 = \frac{(p_1 + p_2)}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$$\boxed{e_2 - e_1 = \frac{p_1 + p_2}{2}(v_1 - v_2)}$$

The above equation is known as Hugonoit equation.



One dimensional flow with heat addition

If heat is added to or taken away from the gas flowing through a duct the properties in region 2 will be different than those in region 1. This is the governing phenomenon in turbojet, ramjet engine burners, where heat is added due to fuel-air combustion.

Consider the one dimensional flow with heat addition between regions 1 and 2. Using the following governing equations the changes are obtained as follows:

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

Solving for q gives

$$q = \left(c_p T_2 + \frac{u_2^2}{2}\right) - \left(c_p T_1 + \frac{u_1^2}{2}\right)$$

From the definition of total temperature, the equation changes to

$$\boxed{q = c_p T_{o_2} - c_p T_{o_1} = c_p(T_{o_2} - T_{o_1})}$$

The above equation clearly states that the effect of heat addition is to directly change total temperature. If heat is added T_o increases and if heat is extracted T_o decreases.

Let us now proceed to ratio of properties

$$\rho u^2 = \rho a^2 M^2 = \rho \frac{\gamma p}{\rho} M^2 = \gamma p M^2$$

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 = \gamma p_1 M_1^2 - \gamma p_2 M_2^2$$

$$\boxed{\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \frac{p_2}{p_1}\frac{u_2}{u_1}$$

$$\frac{u_2}{u_1} = \frac{M_2}{M_1}\frac{a_2}{a_1} = \frac{M_2}{M_1}\left(\frac{T_2}{T_1}\right)^{1/2}$$

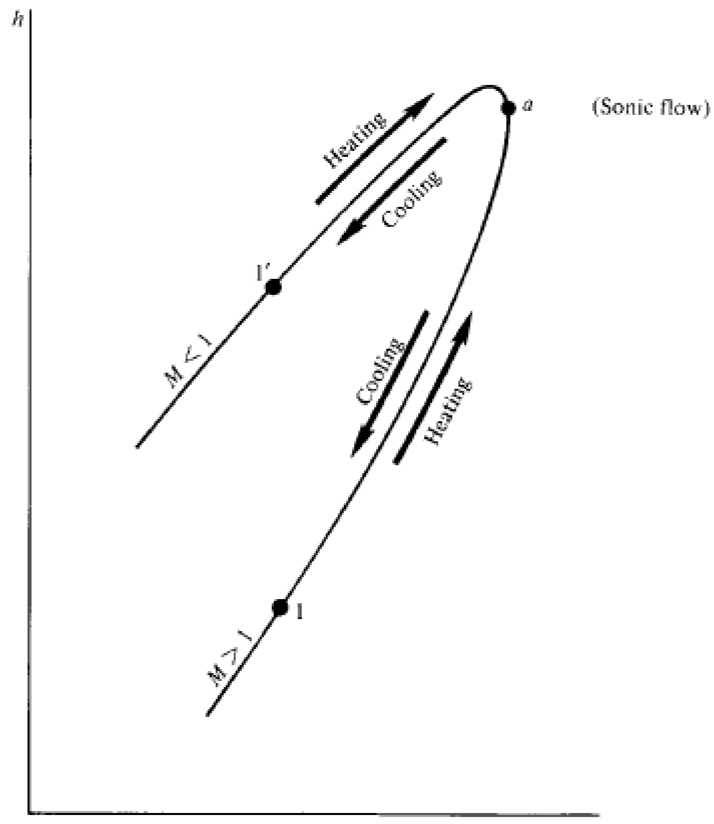
$$\boxed{\frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{1 + \gamma M_2^2}{1 + \gamma M_1^2}\right)\left(\frac{M_1}{M_2}\right)^2}$$

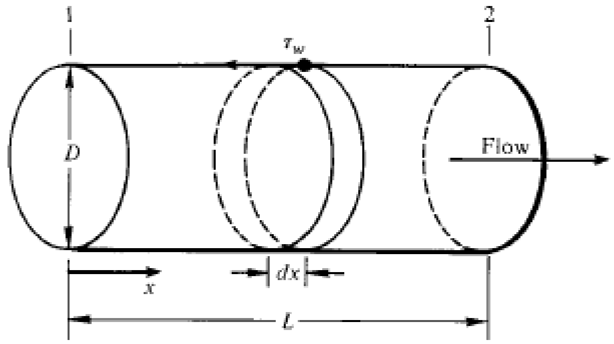
Certain physical trends reflected by the numbers obtained from such solutions are important, and are summarized below:

1. For *supersonic flow* in region 1, i.e., $M_1 > 1$, when heat is added
   a. Mach number decreases, $M_2 < M_1$
   b. Pressure increases, $p_2 > p_1$
   c. Temperature increases, $T_2 > T_1$
   d. Total temperature increases, $T_{o_2} > T_{o_1}$
   e. Total pressure decreases, $p_{o_2} < p_{o_1}$
   f. Velocity decreases, $u_2 < u_1$

2. For *subsonic* flow in region 1, i.e., $M_1 < 1$, when heat is added
   a. Mach number increases, $M_2 > M_1$
   b. Pressure decreases, $p_2 < p_1$
   c. Temperature increases for $M_1 < \gamma^{-1/2}$ and decreases for $M_1 > \gamma^{-1/2}$
   d. Total temperature increases, $T_{o_2} > T_{o_1}$
   e. Total pressure decreases, $p_{o_2} < p_{o_1}$
   f. Velocity increases, $u_2 > u_1$



One dimensional flow with friction



$$\oiint_S (\rho \mathbf{V} \cdot d\mathbf{S})u = -\oiint_S (p\ dS)_x - \oiint_S \tau_w\ dS$$

$$-\rho_1 u_1^2 A + \rho_2 u_2^2 A = p_1 A - p_2 A - \int_0^L \pi D \tau_w\ dx$$

$$(p_2 - p_1) + (\rho_2 u_2^2 - \rho_1 u_1^2) = -\frac{4}{D}\int_0^L \tau_w\ dx$$

$$dp + d(\rho u^2) = -\frac{4}{D}\tau_w\ dx$$

$$dp + \rho u\ du = -\frac{4}{D}\tau_w\ dx$$

$$dp + \rho u\ du = -\tfrac{1}{2}\rho u^2 \frac{4f\ dx}{D}$$

$$\frac{4f\ dx}{D} = \frac{2}{\gamma M^2}(1 - M^2)[1 + \tfrac{1}{2}(\gamma - 1)M^2]^{-1}\frac{dM}{M}$$

$$\boxed{\int_{x_1}^{x_2} \frac{4f\ dx}{D} = \left[-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M^2}{1 + \frac{\gamma-1}{2}M^2}\right)\right]_{M_1}^{M_2}}$$

$$\boxed{\frac{T_2}{T_1} = \frac{T_o/T_1}{T_o/T_2} = \frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}}$$

Also, since $\rho_1 u_1 = \rho_2 u_2$, and $a^2 = \gamma p/\rho$, then

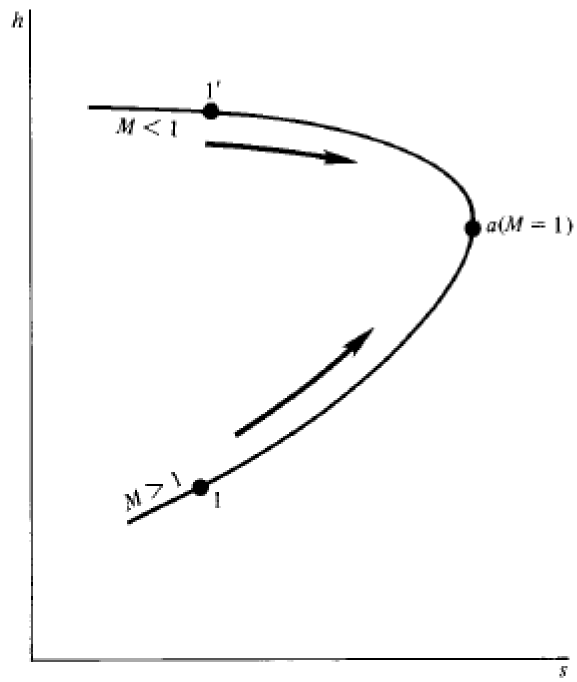
$$\frac{\gamma p_1 u_1}{a_1^2} = \frac{\gamma p_2 u_2}{a_2^2}$$

or

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\frac{a_2}{a_1} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}\right]^{1/2}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}\right]^{-1/2}$$



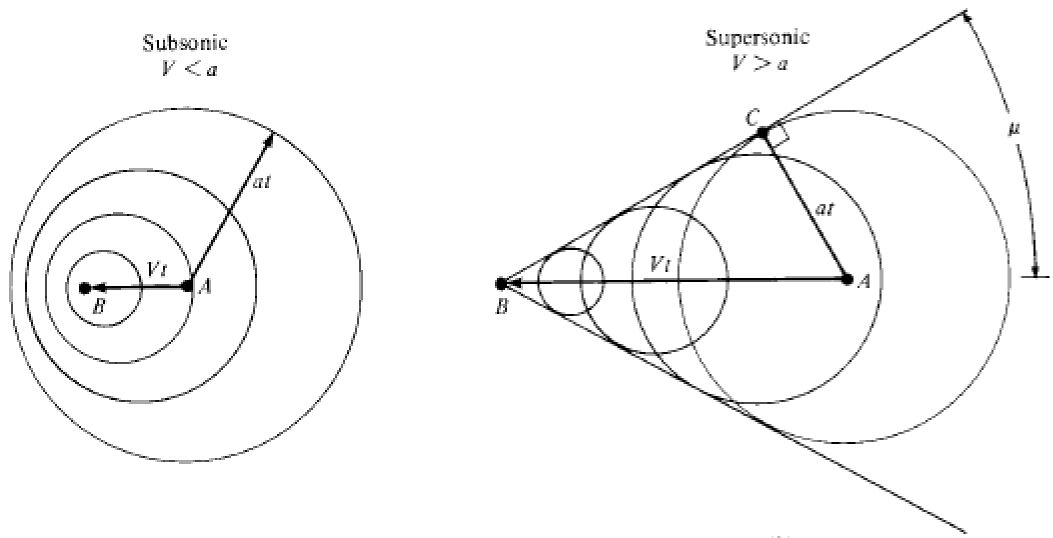
Certain physical trends reflected by the numbers obtained from such solutions are summarized here:

1. For *supersonic* inlet flow, i.e., $M_1 > 1$, the effect of friction on the downstream flow is such that
   a. Mach number decreases, $M_2 < M_1$
   b. Pressure increases, $p_2 > p_1$
   c. Temperature increases, $T_2 > T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity decreases, $u_2 < u_1$
2. For *subsonic* inlet flow, i.e., $M_1 < 1$, the effect of friction on the downstream flow is such that
   a. Mach number increases, $M_2 > M_1$
   b. Pressure decreases, $p_2 < p_1$
   c. Temperature decreases, $T_2 < T_1$
   d. Total pressure decreases, $p_{o_2} < p_{o_1}$
   e. Velocity increases, $u_2 > u_1$

UNIT-II
OBLIQUE SHOCK AND EXPANSION WAVES

When analyzing shock waves in a flow field, which are still attached to the body, the shock wave which is deviating at some arbitrary angle from the flow direction is termed oblique shock. These shocks require a component vector analysis of the flow; doing so allows for the treatment of the flow in an orthogonal direction to the oblique shock as a normal shock.

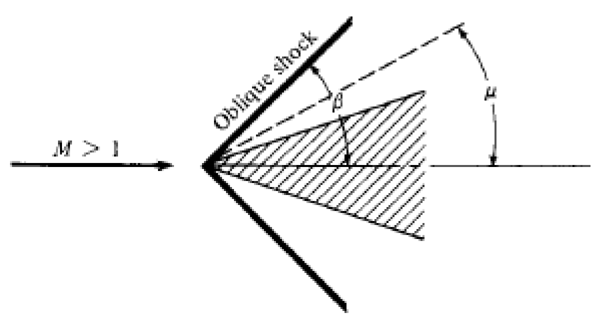
Oblique shock relations



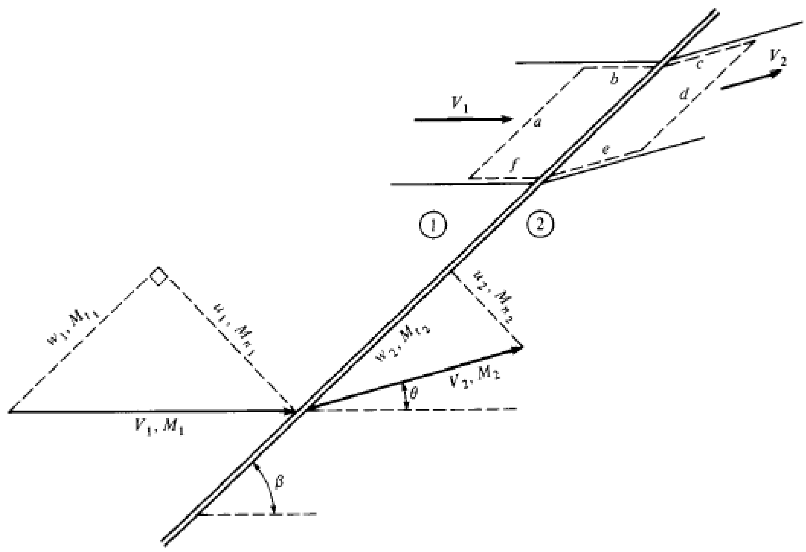
(a) (b)

$$\sin\ \mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$$

$$\boxed{\mu = \sin^{-1}\frac{1}{M}}$$



The geometry of flow through an oblique shock is given in below figure. The velocity upstream of shock is V_1 and is horizontal. The corresponding Mach number is M_1. The oblique shock makes an angle of β with respect to velocity. Behind the shock the flow is deflected toward the shock by the flow-deflection angle θ. The velocity downstream of shock is V_2 and corresponding Mach number is M_2. The parallel and perpendicular velocity components are u and w. Similarly tangential and normal components of Mach are given as M_n and M_t



Applying continuity equation along considered control volume abcde as shown in fig. Areas of faces a and d are same.

$$\rho_1 u_1 = \rho_2 u_2$$

From momentum equation we have

$$(-\rho_1 u_1)w_1 + (\rho_2 u_2)w_2 = 0$$

$$w_1 = w_2$$

This equation shows the tangential component of velocity is preserved across oblique shock. From the geometry of oblique shock we get,

$$M_{n_1} = M_1 \sin \beta$$

we have, for a calorically perfect gas,

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_{n_1}^2}{(\gamma - 1)M_{n_1}^2 + 2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_{n_1}^2 - 1)$$

$$M_{n_2}^2 = \frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_1}^2 - 1}$$

and

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)}$$

Θ - β - M relations strong and weak shock solutions

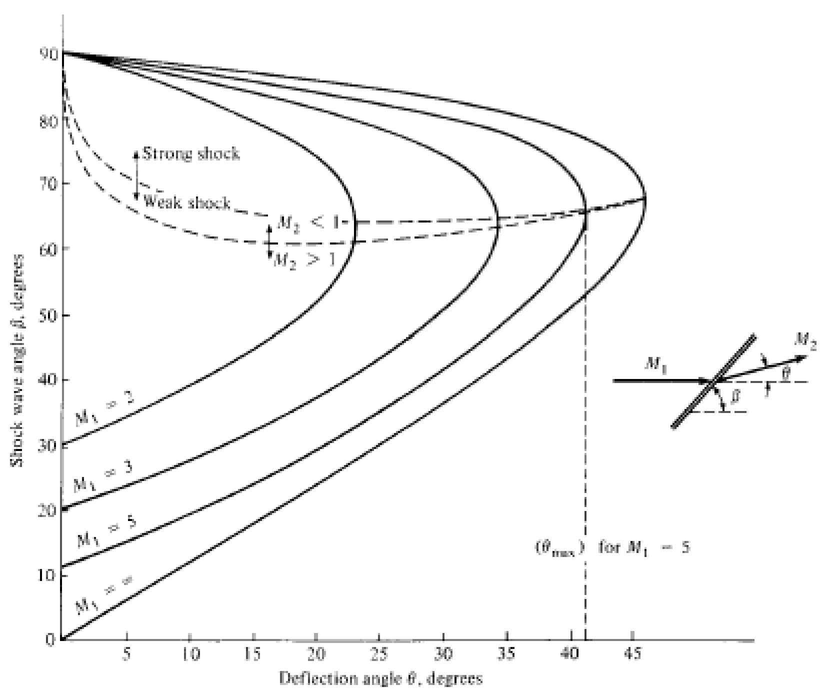
$$\tan\,\beta = \frac{u_1}{w_1}$$

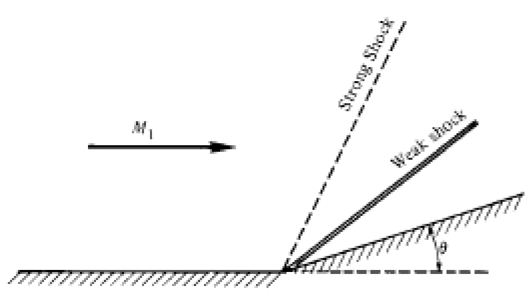
$$\tan\,(\beta - \theta) = \frac{u_2}{w_2}$$

$$\frac{\tan\,(\beta - \theta)}{\tan\,\beta} = \frac{u_2}{u_1}$$

$$\frac{\tan\,(\beta - \theta)}{\tan\,\beta} = \frac{2 + (\gamma - 1)M_1^2\,\sin^2\,\beta}{(\gamma + 1)M_1^2\,\sin^2\,\beta}$$

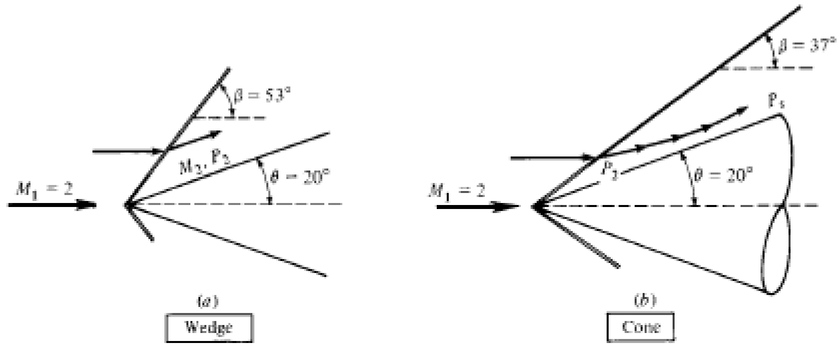
$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos2\beta) + 2}\right]$$





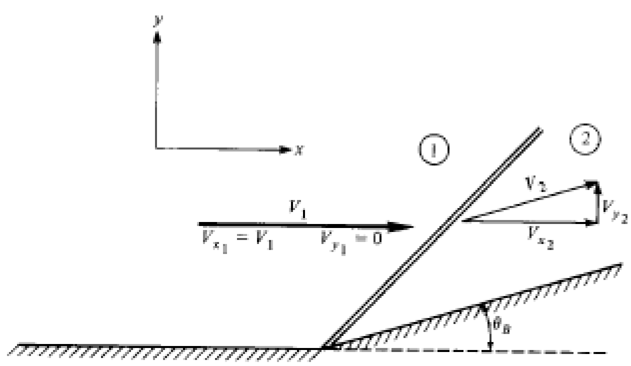
Within the θ-β-M equation, a maximum corner angle, θ_{MAX}, exists for any upstream Mach number. When $\theta > \theta_{MAX}$, the oblique shock wave is no longer attached to the corner and is replaced by a detached bow shock. A θ-β-M diagram, common in most compressible flow textbooks, shows a series of curves that will indicate θ_{MAX} for each Mach number. The θ-β-M relationship will produce two β angles for a given θ and M_1, with the larger angle called a strong shock and the smaller called a weak shock. The weak shock is almost always seen experimentally.

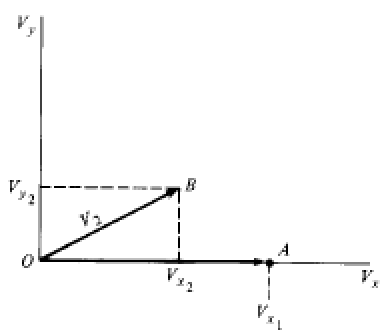
Super sonic flow over a wedge

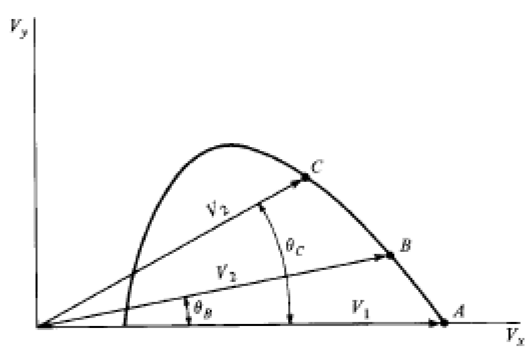


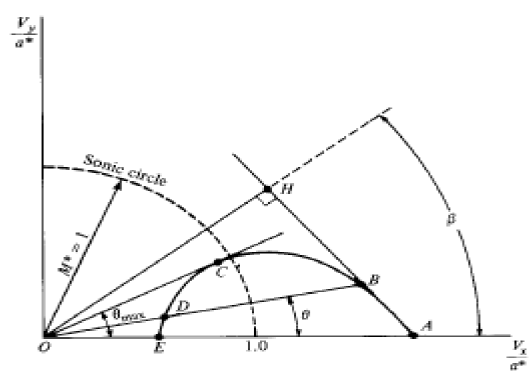
Shock polar

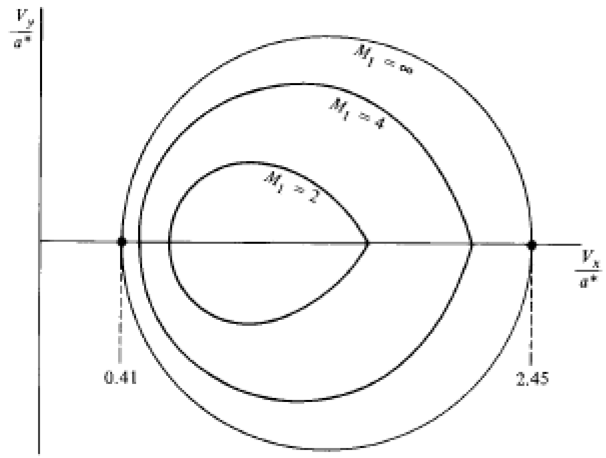
The term *shock polar* is generally used with the graphical representation of the Rankine-Hugoniot equations in either the hodograph plane or the pressure ratio-flow deflection angle plane. The polar itself is the locus of all possible states after an oblique shock.





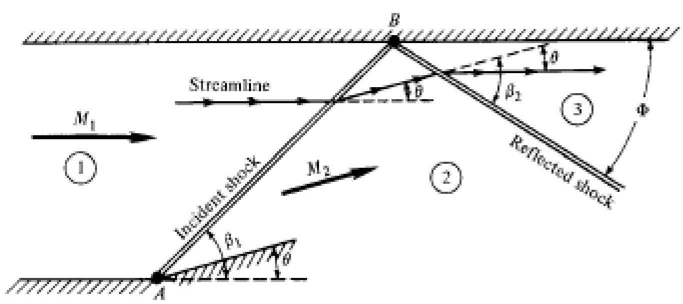






One of the primary uses of shock polars is in the field of shock wave reflection. A shock polar is plotted for the conditions before the incident shock, and a second shock polar is plotted for the conditions behind the shock, with its origin located on the first polar, at the angle through which the incident shock wave deflects the flow. Based on the intersections between the incident shock polar and the reflected shock polar, conclusions as to which reflection patterns are possible may be drawn. Often, it is used to graphically determine whether regular shock reflection is possible, or whether Mach reflection occurs

Regular reflection from a solid boundary

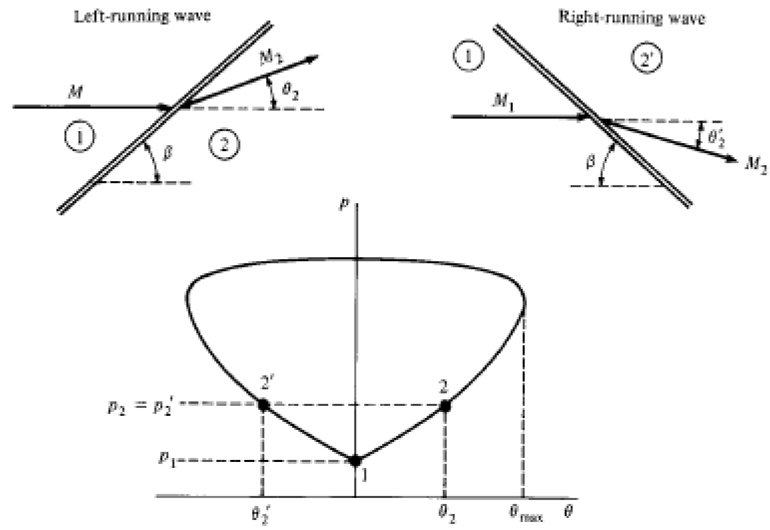


In a steady flow situation, if a wedge is placed into a steady supersonic flow in such a way that its oblique attached shock impinges on a flat wall parallel to the free stream, the shock turns the flow toward the wall and a reflected shock is required to turn the flow back to a direction parallel to the wall. When the shock angle exceeds a certain value, the deflection achievable by a single reflected shock is insufficient to turn the flow back to a direction parallel to the wall and transition to Mach reflection is observed.

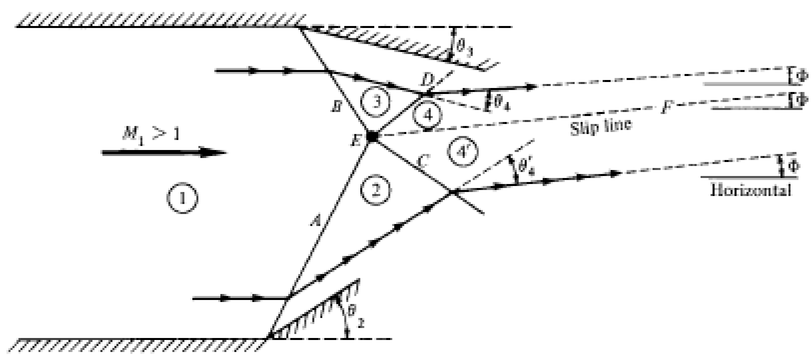
Mach reflection consists of three shocks, namely the incident shock, the reflected shock and a Mach stem, as well as a slip plane. The point where the three shocks meet is known as the 'triple point' in two dimensions, or a shock-shock in three dimensions.

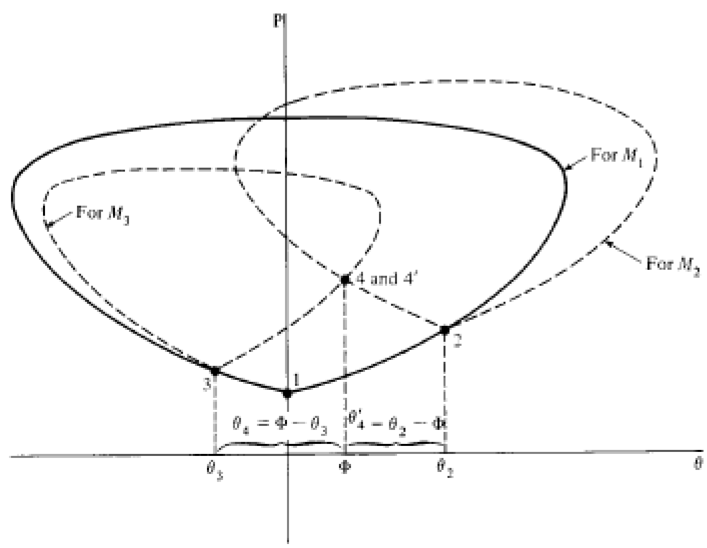
Pressure deflection diagrams

Pressure-deflection diagram is one more tool to present the oblique shock solution. This diagram necessarily plots various possible pressures for change in flow deflection angle for a given Mach number. Hence we can plot this, P-θ diagram, for all the Mach numbers to evaluate the range of pressure for a given Mach number and the same for given deflection angle. This diagram also provides the shock detachment angle. Typical P-θ diagram for a known Mach number is shown shown in **Fig.** All the points marked on this diagram have some special feature associated with them.



Intersections of shock wave

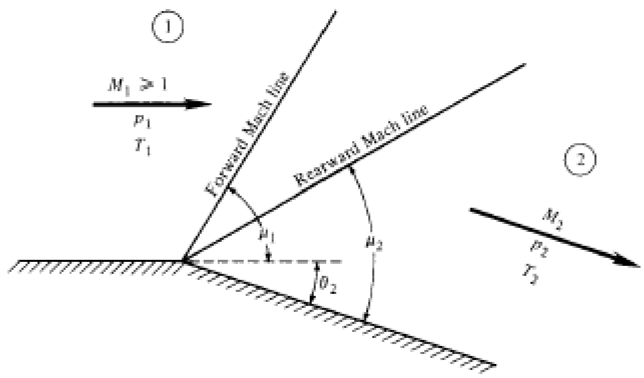




Expansion waves

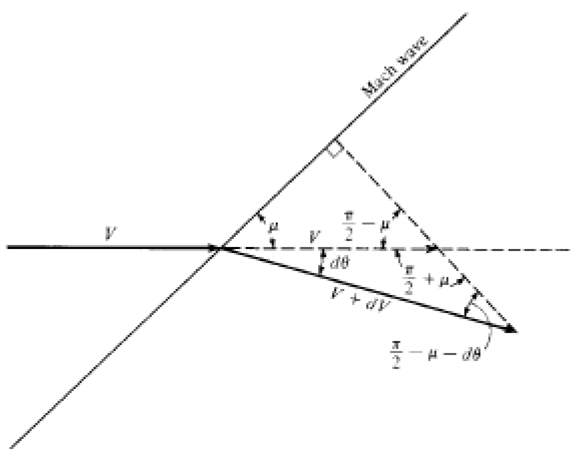
A supersonic expansion fan, technically known as **Prandtl–Meyer expansion fan**, is a centred expansion process that occurs when a supersonic flow turns around a convex corner. The fan consists of an infinite number of Mach waves, diverging from a sharp corner. When a flow turns around a smooth and circular corner, these waves can be extended backwards to meet at a point.

Each wave in the expansion fan turns the flow gradually (in small steps). It is physically impossible for the flow to turn through a single "shock" wave because this would violate the second law of thermodynamics.



Across the expansion fan, the flow accelerates (velocity increases) and the Mach number increases, while the static pressure, temperature and density decrease. Since the process is isentropic, the stagnation properties (e.g. the total pressure and total temperature) remain constant across the fan.

Prandtl – Meyer Expansion



For a given M_1, P_1, T_1 and θ_2 calculating M_2, P_2 and T_2. The analysis can be started by considering the infinitesimal changes across a very weak wave produced by an infinitesimally small flow deflection, $d\theta$. From the law of sines,

$$\frac{V + dV}{V} = \frac{\sin(\pi/2 + \mu)}{\sin(\pi/2 - \mu - d\theta)}$$

However, from trigonometric identities,

$$\sin\left(\frac{\pi}{2} + \mu\right) = \sin\left(\frac{\pi}{2} - \mu\right) = \cos\mu$$

$$\sin\left(\frac{\pi}{2} - \mu - d\theta\right) = \cos(\mu + d\theta) = \cos\mu\cos d\theta - \sin\mu\sin d\theta$$

Solving above equations gives and for small angle dθ, we get

$$1 + \frac{dV}{V} = \frac{\cos \mu}{\cos \mu - d\theta \sin \mu} = \frac{1}{1 - d\theta \tan \mu}$$

Recalling the series expansion (for $x < 1$),

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Then above equation expands to

$$1 + \frac{dV}{V} = 1 + d\theta \tan \mu + \cdots$$

$$d\theta = \frac{dV/V}{\tan \mu}$$

$$\mu = \sin^{-1} \frac{1}{M}$$

$$\tan \mu = \frac{1}{\sqrt{M^2 - 1}}$$

$$\boxed{d\theta = \sqrt{M^2 - 1}\,\frac{dV}{V}}$$

This is the governing differential equation for Prandtl-Meyer flow. To analyze the entire Prandtl-Meyer expansion the above equation must be integrated over the complete angle θ_2 from region 1 to 2

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{M_1}^{M_2} \sqrt{M^2 - 1}\,\frac{dV}{V}$$

From definition of Mach number

$$V = Ma$$

$$\ln V = \ln M + \ln a$$

$$\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a}$$

For a calorically perfect gas, from the alternative forms of energy equation

$$\left(\frac{a_o}{a}\right)^2 = \frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

or, solving for a,

$$a = a_o\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/2}$$

Differentiating above equation,

$$\frac{da}{a} = -\left(\frac{\gamma - 1}{2}\right) M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} dM$$

Solving for dV/V

$$\frac{dV}{V} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2}\,\frac{dM}{M}$$

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - 0 = \int_{M_1}^{M_2} \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}$$

$$v(M) = \int \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}$$

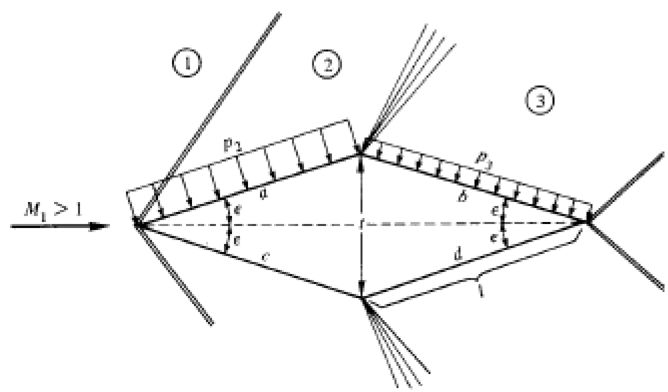
In the above equation the symbol *v* is known as Prandtl-Meyer function and integrating it gives

$$\boxed{v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}}$$

$$\boxed{\theta_2 = v(M_2) - v(M_1)}$$

Shock Expansion theory

The shock and expansion waves discussed in this chapter allow the exact calculation of the aerodynamic force on many types of two-dimensional supersonic airfoils made up of straight line segments. Consider diamond shaped airfoil at zero degrees angle of attack. The supersonic flow first compressed and deflected through the angle ε by an oblique shock wave at the leading edge. At midchord, the flow is expanded through an angle 2ε by the expansion wave. At the trailing edge, the flow is again deflected through the angle ε by another oblique shock; this deflection is necessary to make the flow downstream of the airfoil parallel to the free-stream direction due to symmetric conditions.



At zero degrees angle of attack, the only aerodynamic force on the diamond airfoil will be drag; the lift is zero because the pressure distributions on the top and bottom surfaces are the same.

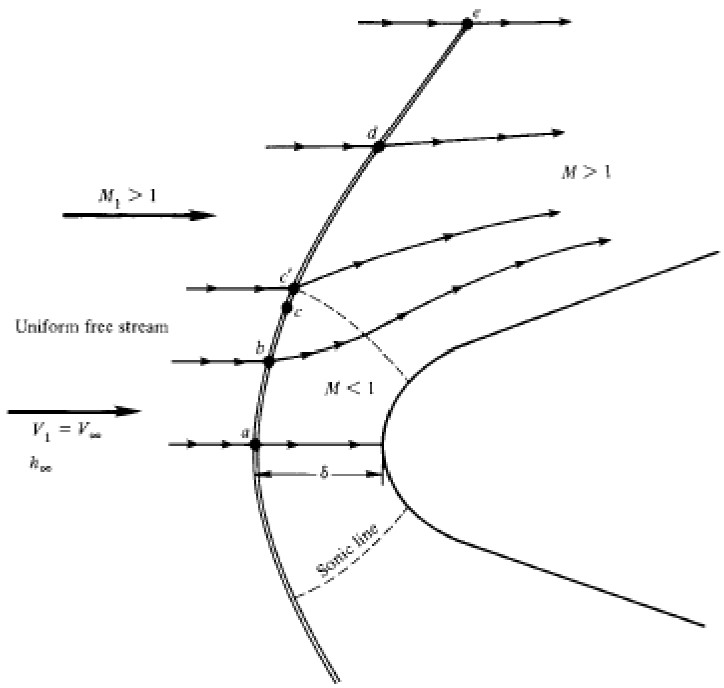
$$D = x \text{ component of } \left[-\oiint p\, d\mathbf{S}\right]$$

$$D = 2(p_2 l \sin\varepsilon - p_3 l \sin\varepsilon) = 2(p_2 - p_3)\frac{t}{2}$$

$$D = (p_2 - p_3)t$$

Detached shock in front of blunt body

In front of a blunt body, generation of oblique shocks is not possible and instead we will get a detached bow shock. In front of the object, the detached shock is normal generating a region of subsonic flow in front of the object. Away from the object, the shock bends of and becomes an oblique shock, which means that the flow behind the shock may very well be supersonic downstream of the shock. This gives us a transonic flow situation, *i.e.* a supersonic flow field with small regions of subsonic flow.


e
d
M > 1
M1 > 1
c'
c
Uniform free stream
b
M < 1
V1 = V∞
h∞
a
δ
Sonic line

UNIT-III
SUBSONIC COMPRESSIBLE AND SUPERSONIC LINEARISED FLOW OVER AIRFOIL

Introduction - **Velocity potential equation**

The general conservation equations derived in previous chapter are simplified for the special case of irrotational flow. It allows the separate continuity, momentum and energy equations with the requisite dependent variables ρ, P, T, V etc to cascade into one governing equation with one dependent variable new defined as velocity potential.

For irrotational flow,∇xV=0. Hence we can define a scalar function ϕ=ϕ(x,y,z) such that

$$\boxed{\mathbf{V} \equiv \nabla\Phi}$$

where Φ is called the *velocity potential*. In cartesian coordinates, since

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

and

$$\nabla\Phi = \frac{\partial\Phi}{\partial x}\mathbf{i} + \frac{\partial\Phi}{\partial y}\mathbf{j} + \frac{\partial\Phi}{\partial z}\mathbf{k}$$

then, by comparison,

$$u = \frac{\partial\Phi}{\partial x} \qquad v = \frac{\partial\Phi}{\partial y} \qquad w = \frac{\partial\Phi}{\partial z}$$

If the velocity potential is known then velocity can be obtained directly from above equations.

$$\nabla \cdot (\rho\mathbf{V}) = 0$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\rho\Phi_x + \frac{\partial}{\partial y}\rho\Phi_y + \frac{\partial}{\partial z}\rho\Phi_z = 0$$

$$\rho(\Phi_{xx} + \Phi_{yy} + \Phi_{zz}) + \Phi_x\frac{\partial\rho}{\partial x} + \Phi_y\frac{\partial\rho}{\partial y} + \Phi_z\frac{\partial\rho}{\partial z} = 0$$

To obtain a complete equation in terms of ϕ eliminate ρ by using Euler's equation.

$$dp = -\rho V\,dV = -\frac{\rho}{2}\,d(V^2) = -\frac{\rho}{2}\,d(u^2 + v^2 + w^2)$$

$$dp = -\rho\,d\left(\frac{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}{2}\right)$$

From speed of sound

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial\rho}\right)_s = a^2$$

$$d\rho = \frac{dp}{a^2}$$

$$d\rho = -\frac{\rho}{a^2}\,d\left(\frac{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}{2}\right)$$

Considering changes in x direction

$$\frac{\partial \rho}{\partial x} = -\frac{\rho}{a^2}\frac{\partial}{\partial x}\left(\frac{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}{2}\right)$$

or

$$\frac{\partial \rho}{\partial x} = -\frac{\rho}{a^2}(\Phi_x\Phi_{xx} + \Phi_y\Phi_{yx} + \Phi_z\Phi_{zx})$$

Similarly,

$$\frac{\partial \rho}{\partial y} = -\frac{\rho}{a^2}(\Phi_x\Phi_{xy} + \Phi_y\Phi_{yy} + \Phi_z\Phi_{zy})$$

$$\frac{\partial \rho}{\partial z} = -\frac{\rho}{a^2}(\Phi_x\Phi_{xz} + \Phi_y\Phi_{yz} + \Phi_z\Phi_{zz})$$

$$\left(1 - \frac{\Phi_x^2}{a^2}\right)\Phi_{xx} + \left(1 - \frac{\Phi_y^2}{a^2}\right)\Phi_{yy} + \left(1 - \frac{\Phi_z^2}{a^2}\right)\Phi_{zz} - \frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy} - \frac{2\Phi_x\Phi_z}{a^2}\Phi_{xz} - \frac{2\Phi_y\Phi_z}{a^2}\Phi_{yz} = 0$$

The above equation is known as velocity potential equation.

Linearized Velocity Potential equation

Consider a slender body immersed in a uniform flow with velocity V_∞ in the x direction. u,v,w denote perturbations from the uniform flow in x,y,z directions respectively. In the perturbed flow pressure, temperature and density are P, T, ρ

$$V_x = V_\infty + u$$
$$V_y = v$$
$$V_z = w$$

In terms of velocity potential

$$\nabla\Phi = \mathbf{V} = (V_\infty + u)\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

Now let us define a new velocity potential, the perturbation velocity potential *φ,* Such that

$$\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial \phi}{\partial y} = v \qquad \frac{\partial \phi}{\partial z} = w$$

$$\Phi(x, y, z) = V_\infty x + \phi(x, y, z)$$

$$V_x = V_\infty + u = \frac{\partial \Phi}{\partial x} = V_\infty + \frac{\partial \phi}{\partial x}$$

$$V_y = v = \frac{\partial \Phi}{\partial y} = \frac{\partial \phi}{\partial y}$$

$$V_z = w = \frac{\partial \Phi}{\partial z} = \frac{\partial \phi}{\partial z}$$

Consider again the velocity potential equation multiplying with a^2 and substituting $\phi = V_\infty x + \varphi$

$$\left[a^2 - \left(V_\infty + \frac{\partial \phi}{\partial x}\right)^2\right]\frac{\partial^2 \phi}{\partial x^2} + \left[a^2 - \left(\frac{\partial \phi}{\partial y}\right)^2\right]\frac{\partial^2 \phi}{\partial y^2} + \left[a^2 - \left(\frac{\partial \phi}{\partial z}\right)^2\right]\frac{\partial^2 \phi}{\partial z^2} - 2\left(V_\infty + \frac{\partial \phi}{\partial x}\right)\frac{\partial \phi}{\partial y}\frac{\partial^2 \phi}{\partial x \partial y} - 2\left(V_\infty + \frac{\partial \phi}{\partial x}\right)\frac{\partial \phi}{\partial z}\frac{\partial^2 \phi}{\partial x \partial z} - 2\frac{\partial \phi}{\partial y}\frac{\partial \phi}{\partial z}\frac{\partial^2 \phi}{\partial y \partial z} = 0$$

This equation is known as perturbation-velocity potential equation.

Since total enthalpy is constant throughout the flow,

$$h_\infty + \frac{V_\infty^2}{2} = h + \frac{V^2}{2} = h + \frac{(V_\infty + u)^2 + v^2 + w^2}{2}$$

$$\frac{a_\infty^2}{\gamma - 1} + \frac{V_\infty^2}{2} = \frac{a^2}{\gamma - 1} + \frac{(V_\infty + u)^2 + v^2 + w^2}{2}$$

$$a^2 = a_\infty^2 - \frac{\gamma - 1}{2}(2uV_\infty + u^2 + v^2 + w^2)$$

Substituting above equation in perturbation velocity potential equation and rearranging algebraically

$$\begin{aligned}(1 - M_\infty^2)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \\ = M_\infty^2\left[(\gamma + 1)\frac{u}{V_\infty} + \left(\frac{\gamma + 1}{2}\right)\frac{u^2}{V_\infty^2} + \left(\frac{\gamma - 1}{2}\right)\left(\frac{v^2 + w^2}{V_\infty^2}\right)\right]\frac{\partial u}{\partial x} \\ + M_\infty^2\left[(\gamma - 1)\frac{u}{V_\infty} + \left(\frac{\gamma + 1}{2}\right)\frac{v^2}{V_\infty^2} + \left(\frac{\gamma - 1}{2}\right)\left(\frac{w^2 + u^2}{V_\infty^2}\right)\right]\frac{\partial v}{\partial y} \\ + M_\infty^2\left[(\gamma - 1)\frac{u}{V_\infty} + \left(\frac{\gamma + 1}{2}\right)\frac{w^2}{V_\infty^2} + \left(\frac{\gamma - 1}{2}\right)\left(\frac{u^2 + v^2}{V_\infty^2}\right)\right]\frac{\partial w}{\partial z} \\ + M_\infty^2\left[\frac{v}{V_\infty}\left(1 + \frac{u}{V_\infty}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{w}{V_\infty}\left(1 + \frac{u}{V_\infty}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \frac{vw}{V_\infty^2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right]\end{aligned}$$

Consider small perturbations, i.e., u,v,w are small compared to free stream velocity.

$$\frac{u}{V_\infty}, \frac{v}{V_\infty}, \text{ and } \frac{w}{V_\infty} \ll 1 \qquad \left(\frac{u}{V_\infty}\right)^2, \left(\frac{v}{V_\infty}\right)^2, \text{ and } \left(\frac{w}{V_\infty}\right)^2 \lll 1$$

For subsonic flows and for supersonic flows, the magnitude

$$M_\infty^2\left[(\gamma + 1)\frac{u}{V_\infty} + \cdots\right]\frac{\partial u}{\partial x}$$

is small in comparison to the magnitude of

$$(1 - M_\infty^2)\frac{\partial u}{\partial x}$$

Thus, *ignore* the former term.

For supersonic flows Mach < 5

$$M_\infty^2\left[(\gamma - 1)\frac{u}{V_\infty} + \cdots\right]\frac{\partial v}{\partial y}$$

s small in comparison to $\partial v/\partial y$,

$$M_\infty^2\left[(\gamma - 1)\frac{u}{V} + \cdots\right]\frac{\partial w}{\partial z}$$

s small in comparison to $\partial w/\partial z$, and

$$M_\infty^2\left[\frac{v}{V_\infty}\left(1 + \frac{u}{V_\infty}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \cdots\right] \approx 0$$

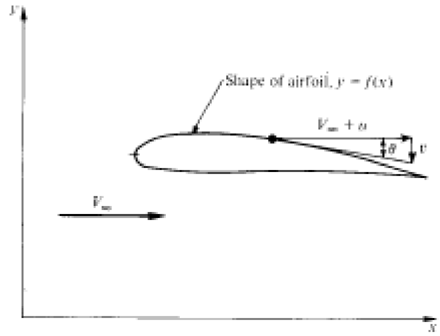
$$\boxed{(1 - M_\infty^2)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

This equation is known as linearized perturbation velocity potential equation.

Prandtl-Glauert compressibility corrections

Consider the compressible subsonic flow over a thin airfoil at small angle of attack (hence small perturbations). The usual inviscid flow boundary condition must hold at the surface, i.e., the flow velocity must be tangent to the surface.

$$\frac{df}{dx} = \frac{v}{V_\infty + u} = \tan\theta$$



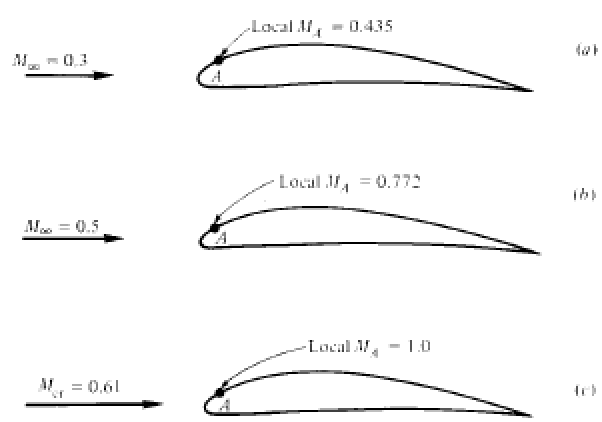
$$C_p = \frac{C_{p_0}}{\sqrt{1 - M_\infty^2}}$$

$$C_L = \frac{C_{L_0}}{\sqrt{1 - M_\infty^2}}$$

$$C_M = \frac{C_{M_0}}{\sqrt{1 - M_\infty^2}}$$

Critical Mach number

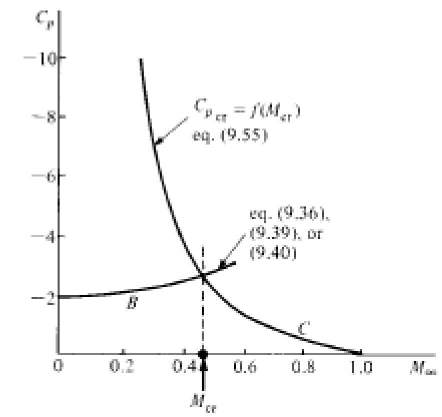
At high-subsonic flight speeds, the local speed of the airflow can reach the speed of sound where the flow accelerates around the aircraft body and wings. The speed at which this development occurs varies from aircraft to aircraft and is known as the critical Mach number. The resulting shock waves formed at these points of sonic flow can greatly reduce power, which is experienced by the aircraft as a sudden and very powerful drag, called wave drag. To reduce the number and power of these shock waves, an aerodynamic shape should change in cross sectional area as smoothly as possible.



$$\frac{p_A}{p_\infty} = \left(\frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{1 + \frac{\gamma - 1}{2} M_A^2} \right)^{\gamma/(\gamma - 1)}$$

$$C_{p_A} = \frac{2}{\gamma M_\infty^2} \left[\left(\frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{1 + \frac{\gamma - 1}{2} M_A^2} \right)^{\gamma/(\gamma - 1)} - 1 \right]$$

$$C_{p_{cr}} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1 + \frac{\gamma - 1}{2} M_{cr}^2}{1 + \frac{\gamma - 1}{2}}\right)^{\gamma/(\gamma - 1)} - 1\right]$$



Drag divergence Mach number

The **drag divergence Mach number** (not to be confused with critical Mach number) is the Mach number at which the aerodynamic drag on an airfoil or airframe begins to increase rapidly as the Mach number continues to increase.[1] This increase can cause the drag coefficient to rise to more than ten times its low speed value.

The value of the drag divergence Mach number is typically greater than 0.6; therefore it is a transonic effect. The drag divergence Mach number is usually close to, and always greater than, the critical Mach number. Generally, the drag coefficient peaks at Mach 1.0 and begins to decrease again after the transition into the supersonic regime above approximately Mach 1.2.

The large increase in drag is caused by the formation of a shock wave on the upper surface of the airfoil, which can induce flow separation and adverse pressure gradients on the aft portion of the wing. This effect requires that aircraft intended to fly at supersonic speeds have a large amount of thrust. In early development of transonic and supersonic aircraft, a steep dive was often used to provide extra acceleration through the high drag region around Mach 1.0. This steep increase in drag gave rise to the popular false notion of an unbreakable sound barrier, because it seemed that no aircraft technology in the foreseeable future would have enough propulsive force or control authority to overcome it. Indeed, one of the popular analytical methods for calculating drag at high speeds, the Prandtl-Glauert rule, predicts an infinite amount of drag at Mach 1.0.

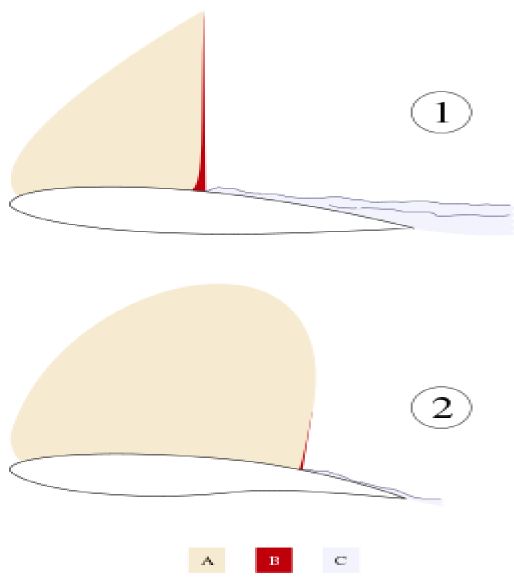
Two of the important technological advancements that arose out of attempts to conquer the sound barrier were the Whitcomb area rule and the supercritical airfoil. A supercritical airfoil is shaped specifically to make the drag divergence Mach number as high as possible, allowing aircraft to fly with relatively lower drag at high subsonic and low transonic speeds.

Area rule

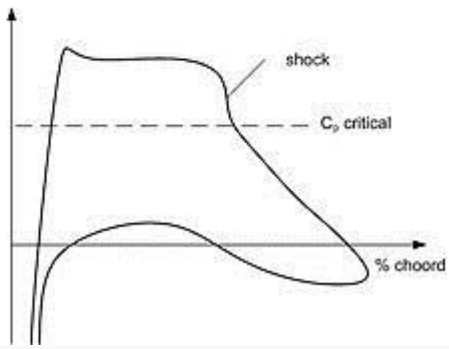
The **Whitcomb area rule**, also called the **transonic area rule**, is a design technique used to reduce an aircraft's drag at transonic and supersonic speeds, particularly between Mach 0.75 and 1.2. The area rule says that two airplanes with the same longitudinal cross-sectional area distribution have the same wave drag, independent of how the area is distributed laterally (i.e. in the fuselage or in the wing). Furthermore, to avoid the formation of strong shock waves, this total area distribution must be smooth. As a result, aircraft have to be carefully arranged so that at the location of the wing, the fuselage is narrowed or "waisted", so that the total area does not change much. Similar but less pronounced fuselage waisting is used at the location of a bubble canopy and perhaps the tail surfaces.

Supercritical airfoil

A **supercritical airfoil** is an airfoil designed, primarily, to delay the onset of wave drag in the transonic speed range. Supercritical airfoils are characterized by their flattened upper surface, highly cambered ("downward-curved") aft section, and larger leading edge radius compared with NACA 6-series laminar airfoil shapes. Standard wing shapes are designed to create lower pressure over the top of the wing. The camber of the wing determines how much the air accelerates around the wing. As the speed of the aircraft approaches the speed of sound the air accelerating around the wing will reach Mach 1 and shockwaves will begin to form. The formation of these shockwaves causes wave drag. Supercritical airfoils are designed to minimize this effect by flattening the upper surface of the wing.



Conventional (1) and supercritical (2) airfoils at identical free stream Mach number. Illustrated are: A, Supersonic flow region; B, Shock wave; C, Area of separated flow. The supersonic flow over a supercritical airfoil terminates in a weaker shock, thereby postponing shock-induced boundary layer separation.



Supercritical airfoil Mach Number/pressure coefficient diagram. The sudden increase in pressure coefficient at midchord is due to the shock. (y-axis:Mach number (or pressure coefficient, negative up); x-axis: position along chord, leading edge left)

Improved compressibility correction factors,

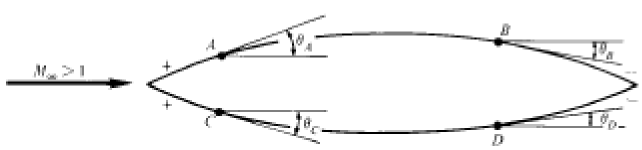
The Karman-Tsien rule states

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2} + [M_\infty^2/(1 + \sqrt{1 - M_\infty^2})]C_{p,0}/2}$$

Laitone's rule states

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2} + (M_\infty^2[1 + [(\gamma - 1)/2]M_\infty^2]/2\sqrt{1 - M_\infty^2})C_{p,0}}$$

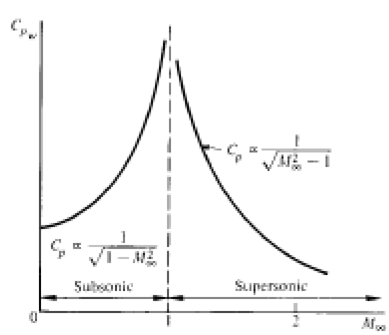
Application to airfoil



Biconvex airfoil

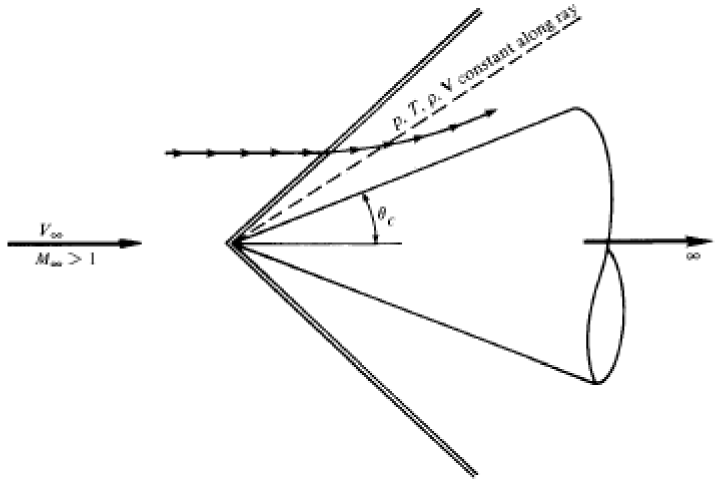
At two arbitrary points A and B on the top surface,

$$C_{p_A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p_B} = \frac{2\theta_B}{\sqrt{M_\infty^2 - 1}}$$



Conical flows-physical aspects

The flow over a cone is a two-dimensional axisymmetric problem. It is also referred to as "Quasi-Two dimensional Problem". This is so, because, the cone under consideration is aligned symmetrically about the z-axis or along the direction of V_∞ , as shown in the **Fig**



The supersonic flow over a cone is of great practical importance in applied aerodynamics. The nose cones of many high-speed missiles and projectiles are approximately conical, are the nose regions of the fuselages of most supersonic airplanes.

In the particular problem of supersonic flow over a cone, consideration is given to a sharp right circular cone with zero angle of attack. Consider a cone on the (r, ϕ, z) co-ordinate system, as shown in **Fig.** which is symmetric about the Z axis and extends to infinity with a semi-vertex cone angle ϑ. The supersonic flow with freestream velocity V_∞ is considered along the Z axis, such that the angle of attack is 0°. Tytpical flowfield for supersonic flow over cone is as shown in **Fig. 30.2**. For such a supersonic flow over the surface of the cone, it is expected that a oblique shock wave attached to the tip of the cone is formed. Further, the shape of the shock wave formed is also conical. A streamline from the supersonic freestream discontinuously deflects as it passes through the shock, and then curves continuously downstream of the shock, becoming parallel to the cone surface asymptotically at infinity. Further, it is also assumed that the pressure and all the other flow properties are constant along the surface of the cone. Since the cone surface is simply a ray from the vertex, consider other such rays between the cone surface and the shock wave. Hence assumption of constacny of flow properties can be extended along these rays as well. therefore properties variation takes place as the fluid moves from one ray to the next.

Delta Wing Aerodynamics

The **delta wing** is a wing shaped in the form of a triangle. It is named for its similarity in shape to the Greek uppercase letter delta (Δ).

General characteristics

The long root chord and short span of the delta wing make it structurally efficient. It can be built stronger, stiffer and at the same time lighter than a swept wing of equivalent lifting capability. Its long root chord also allows a deeper structure for a given aerofoil section, providing more internal volume for fuel and other storage. Because of its light, robust structure it is easy and relatively inexpensive to build – a substantial factor in the success of the MiG-21 and Mirage aircraft.

The tailless delta wing is not suited to high wing loadings and requires a large wing area for a given aircraft weight. The most efficient aerofoils are unstable in pitch and the tailless type must use a less efficient design and therefore a bigger wing. Techniques used include:

- Using a less efficient aerofoil which is inherently stable, such as a symmetrical form with zero camber, or even reflex camber near the trailing edge,
- Using the rear part of the wing as a lightly- or even negatively-loaded horizontal stabiliser:
- Twisting the outer leading edge down to reduce the incidence of the wing tip, which is behind the main centre of lift. This also improves stall characteristics and can benefit supersonic cruise in other ways.
- Moving the centre of mass forwards and trimming the elevator to exert a balancing downforce. In the extreme, this reduces the craft's ability to pitch its nose up for takeoff and landing.

Low-speed characteristics

The Eurofighter Typhoon of the German Air Force has a tailless delta wing configuration.

At low speeds a delta wing requires a high angle of attack to maintain lift. A slender delta creates a characteristic vortex pattern over the upper surface which enhances lift. Some types with intermediate sweep have been given retractable "moustaches" or fixed leading-edge root extensions (LERX) to encourage vortex formation.

As the angle of attack increases, the leading edge of the wing generates a vortex which energizes the flow on the upper surface of the wing, delaying flow separation, and giving the delta a very high stall angle.[*citation needed*] A normal wing built for high speed use typically has undesirable characteristics at low speeds, but in this regime the delta gradually changes over to a mode of lift based on the vortex it generates, a mode where it has smooth and stable flight characteristics.

The vortex lift comes at the cost of increased drag, so more powerful engines are needed to maintain low speed or high angle-of-attack flight.

Transonic and supersonic characteristics

Wind tunnel model of the Avro 720, a delta-winged aircraft, in The Science Museum's Blythe House store

With a large enough angle of rearward sweep, in the transonic to low supersonic speed range the wing's leading edge remains behind the shock wave boundary or shock cone created by the leading edge root.

This allows air below the leading edge to flow out, up and around it, then back inwards creating a sideways flow pattern. The lift distribution and other aerodynamic characteristics are strongly influenced by this sideways flow.[14]

The rearward sweep angle lowers the airspeed normal to the leading edge of the wing, thereby allowing the aircraft to fly at high subsonic, transonic, or supersonic speed, while the subsonic lifting characteristics of the airflow over the wing are maintained.

Within this flight regime, drooping the leading edge within the shock cone increases lift but not drag.[15] Such conical leading edge droop was introduced on the production Convair F-102A Delta Dagger at the same time that the prototype design was reworked to include area-ruling. It also appeared on Convair's next two deltas, the F-106 Delta Dart and B-58 Hustler.[16]

At high supersonic speeds the shock cone from the leading edge root angles further back to lie along the wing surface behind the leading edge. It is no longer possible for the sideways flow to occur and the aerodynamic characteristics change considerably.[14] It is in this flight regime that the waverider technique, as used on the North American XB-70 Valkyrie, becomes practicable. Here, a shock body beneath the wing creates an attached shockwave and the high pressure associated with the wave provides significant lift without increasing drag.